7.2

Working the System

Systems of Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Write and graph systems of linear inequalities.
- Determine solutions to systems of linear inequalities.
- Algebraically prove solutions and non-solutions of systems of linear inequalities.
- Graph systems of linear inequalities using a graphing calculator.

KEY TERMS

- constraints
- solution of a system of linear inequalities

Whitewater rafting is a challenging outdoor activity. It involves navigating through a river or other body of water in an inflatable raft. There are 6 different grades of difficulty in whitewater rafting based on the speed of the current and the hazards rafters may encounter. Grade 1 rafting involves very few rough areas that require some maneuvering of the raft. Grade 1 rafting is good for beginners or children. Grade 6 rafting is so dangerous that there may be times when the waterway is impassable. Rafters can expect to see huge waves and rocks as well as substantial drops. Grade 6 rafting can actually be deadly! However, by using the proper safety gear and traveling with a reliable guide, thousands of people safely enjoy rafting trips every year!

Whitewater rafting often involves a number of people rafting together. Do you think having more or fewer people in the raft would make the trip safer or more dangerous? What else might affect the safety of the raft?

PROBLEM 1 Whitewater Rafting

Chase is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The super-raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds will cause the raft to sink, hit more rocks, and maneuver more slowly.

1. Chase estimates the weight of each adult as approximately 200 pounds and the weight of each child under age sixteen as approximately 100 pounds. Chase charges adults $75 and children under age sixteen $50 to ride down the river with him. His goal is to earn at least $150 each rafting trip.

   a. Write an inequality to represent the most weight Chase can carry in terms of rafters. Define your variables.

   b. Write an inequality to represent the least amount of money Chase wants to collect for each rafting trip.

   c. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Chase wants to earn per trip.

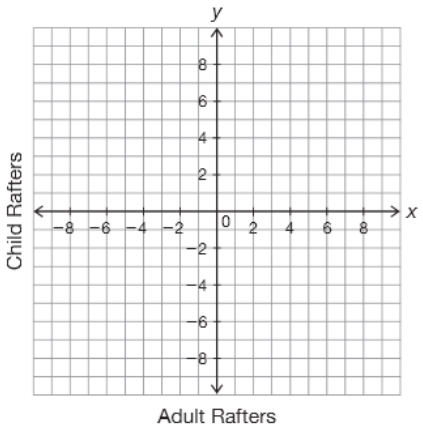
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

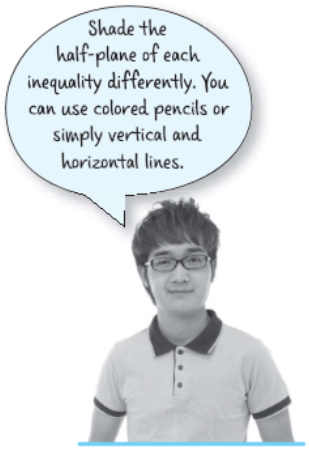
2. Let's consider the past two trips that Chase guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then describe the meaning of the solution in terms of this problem situation.

   a. First Trip: Chase guides 2 adults and 2 children.

b. Second Trip: Chase guides 5 adults.

3. Graph the system of linear inequalities on the coordinate plane shown.





The **solution of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies the solution.

4. Analyze your graph.

a. Describe the possible number of solutions for a system of linear inequalities.

b. Is the intersection point a solution to this system of inequalities? Why or why not?

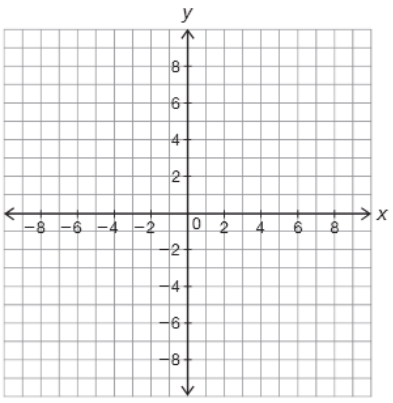
c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?

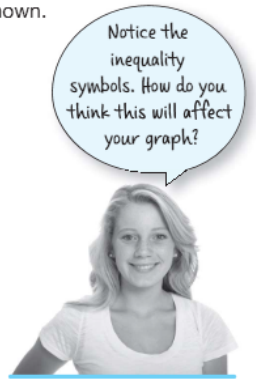
d. Determine one combination of adults and children that is *not* a solution for this system of linear inequalities. Explain your reasoning.

5. Analyze the solution set of the system of linear inequalities shown.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

a. Graph the system of linear inequalities.





b. Choose a point in each shaded region of the graph. Determine whether each point is a solution of the system. Then describe how the shaded region represents the solution.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
$(-8, 2)$	$-8 + 2 > 1$ $-6 > 1$ ✗	$-(-8) + 2 \leq 3$ $10 \leq 3$ ✗	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.

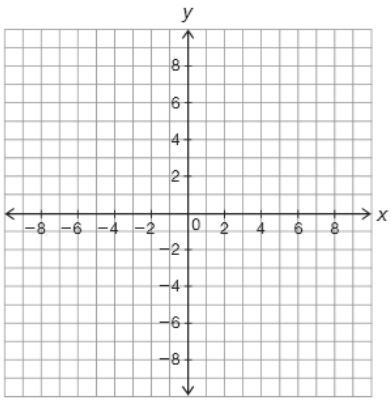
c. Alan makes the statement shown.

> **Alan**
>
> The intersection point is always an algebraic solution to a system of inequalities because that is where the two lines meet.

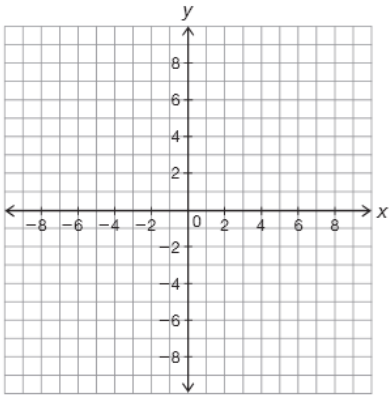
Explain why Alan's statement is incorrect. Use the intersection point of this system to explain your reasoning.

6. Solve each system of linear inequalities by graphing the solution set. Then identify two points that are solutions of the system.

a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$



b. $\begin{cases} x \geq -4 \\ x \geq 1 \end{cases}$



PROBLEM 2 Burning Calories

Jackson and a group of friends decide to use the fitness room after school. On the wall, they read the information shown:

Exercise	Calories Burned per Minute
Treadmill—light effort	7.6
Treadmill—vigorous effort	12.4
Stair Stepper—light effort	6.9
Stair Stepper—vigorous effort	10.4
Stationary Bike—light effort	5.5
Stationary Bike—vigorous effort	11.1

Jackson decides to use the stair stepper. He has at most 45 minutes to exercise and he wants to burn at least 400 calories.

1. Write a system of linear inequalities to represent Jackson's workout. Define your variables.

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Let's graph the system you wrote in Question 1.

You can use a graphing calculator to graph a system of linear inequalities.

Step 1: Press **Y=** and enter the two inequalities as Y_1 and Y_2.

Step 2: While still in the **Y=** window, access the inequality function by moving your cursor to the left until the \ flashes. Press **ENTER** to select the appropriate inequality symbol (or).

Step 3: Press **WINDOW** and set the bounds.

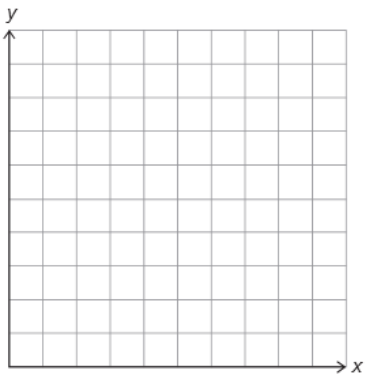
Step 4: Press **GRAPH**.

When choosing the inequality symbol, think about the half-plane you must shade.

Remember to solve for the y-value before entering the inequalities.

Set the WINDOW for this problem using the bounds [0, 50] X [0, 50].

2. Graph the system of inequalities from Question 1 on the coordinate plane shown. Be sure to label your axes.



3. Analyze your graph.

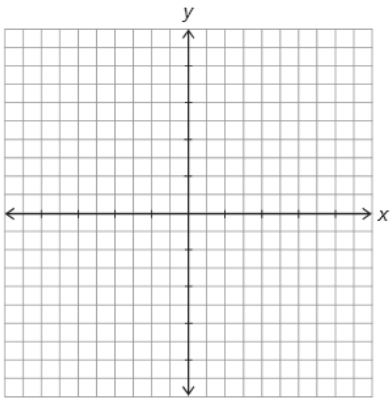
   a. Identify two different solutions of the system of inequalities using the **value** function of your graphing calculator.

   b. Interpret your solutions in terms of Jackson's workout.

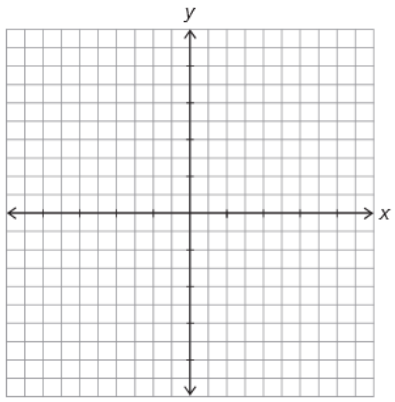
   c. Algebraically prove that your solutions satisfy the system of linear inequalities.

4. Solve each system of linear inequalities using your graphing calculator. Graph each system then identify two points that are solutions to the system on the grid shown.

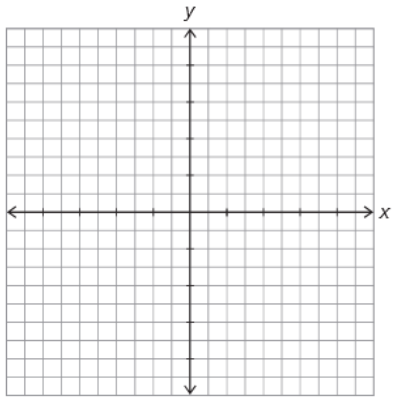
   a. $\begin{cases} y < \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$



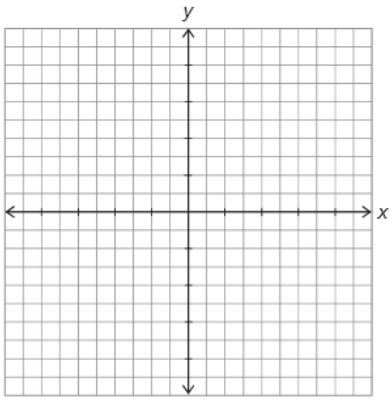
b. $\begin{cases} y > \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$



c. $\begin{cases} y > \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$



d. $\begin{cases} y < \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$



5. Adele states that since the equations in each system for Question 4 are the same, the graphs and solutions should all be identical. Is Adele's statement true? Explain your reasoning.

Be prepared to share your solutions and methods.

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